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The Theoretic Research of Tachyons with Real Mass: Tachyon Transformation Matrix, Tachyon Oscillations, and Measuring Tachyon Velocity

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Abstract: In this paper, a theoretical approach has been used in order to present the physical characteristics of tachyons with real mass. The procedure of the transfer from the Einstein's physics into the world of superluminal particles has been given. In addition, the tachyon transformation matrix has been constructed using the principle of correspondence between these two physics. With the usage of the tachyon matrix, it has been shown how length contraction and time dilatation are calculated in the tachyons case. A particular attention has been devoted to measuring the velocity of tachyons and their potential flavor oscillations since it should be kept in mind that there is no rest reference frame attached to tachyon world lines and, in that sense, special relativity does not treat tachyons on the same footing as particles that are slower than light. It has been demonstrated, using the Lorentz transformation matrix, that it is impossible to measure the velocities above the speed of light with the method of measuring time of flight in laboratories over a certain distance. It has been particularly disclosed that tachyons as isolated particles each on its own could not exist in nature, and if they did exist, they would always appear united with other tachyon types. Owing to that tachyon characteristic, obtained by theoretical consideration, it has been concluded that tachyons, rather than neutrinos as subluminal particles, could comply with the definition for the occurrence of the oscillation phenomena between different tachyon types. Furthermore, the analysis of the velocity of emitted neutrinos during the explosion of Supernova SN1987A has been conducted in the spirit of the proposed theory, where it has been demonstrated that it is possible to measure even superluminal velocities with the usage of that measuring method.

Keywords: Special Relativity, Leptons, Tachyons, Neutrino Mass and Mixing

1. Introduction

At first, after the discovery of neutrinos, it was considered that they possess no mass, which was the reason of claims that their velocity coincides with the speed of light. Further development of the physics of neutrinos experimentally established their feature of turning from one flavor state into another flavor state and vice versa, in the process known as the neutrino oscillation [1-3].

Neutrino oscillation confirmed that neutrinos possess mass and that changed the understanding of their velocity. Mass no longer equals zero, it exists, although it has a rather low, still undefined value. Due to the possession of such mass, contemporary physics considers that the neutrino velocity would be a little lower than the speed of light. Whether the existence of mass is the main reason for such a neutrino velocity or there is something else in question, remains to be seen in further consideration of the proposed theory of superluminal particles.

All measurements of the neutrino velocity conducted in laboratories, regardless of the location of those measurements, showed almost identical results [4-8], the value of which is a little below the speed of light. Such a small deviation from the speed of light would be attributed to their low mass that is still a mysterious and unresolved value in physics.

G. Feinberg was one of the first to propose the theory of particles faster than light as early as in 1967. [9]. Since he based his theory on the grounds of Einstein's formula for the energy of a particle, which was well known in physics, it resulted in the fact that the mass of a particle which they called tachyon had to be an imaginary value. It was simply computed by taking a square root of a negative number. Thus, the basis of all the previous theories in physics related to a tachyon as a theoretically postulated particle which moved faster than light, but which possessed imaginary mass.

Contrary to previous theories, this paper underlines that there is an essential difference between the name of particles faster than light which was introduced in the previous theories and a tachyon which is the subject of the research in this work.

Namely, in previous theories, a tachyon was observed as a particle with imaginary mass, while, in the given theoretical model of this work, a tachyon is considered as a particle which possesses real mass.

That is why it must be stated that a tachyon as a particle with imaginary mass does not belong to the real world of particles.

That is actually the only reason to search for a possible mathematical model which would describe a tachyon as a particle with real mass, and give it a chance to join the world of real particles. Such a mathematical model had to arise from Einstein's energy relation where the limit determined by the speed of light would be easily skipped and, thus, a domain defined by the speed of motion would be extended to infinite velocity.

Additionally, it is generally known that in today's physics, tachyons represent only hypothetical particles which have not been measured in real experiments, so far.

However, regardless of such a status of a tachyon in today's physics, this paper considers, for now, that there is at least a theoretical possibility for the existence of a tachyon.

According to the above, the similar attitude states that there are some suggestions given in the reference [10, 16], where it is considered that neutrinos could be tachyons.

In order to make a comparison between the previous theories and the one suggested in this work, it will be shown how to make a transition from Einstein's physics, defined by the top-limit speed being equal to the speed of light and the physics of tachyons with real mass, where the domain of speed is extended from the speed of light to an infinite value. A special chapter is dedicated to the defining of spacetime fabric and its application in the definition of the velocity of tachyons.

The main idea and the procedure of defining energy function of a tachyon with real mass are also given. At that point, the transition from Einstein's physics to the physics of superluminal particles is presented.

By defining the energy function of a tachyon with real mass, there is a possibility to introduce not one, but two energy functions, which will be done in the next section.

The contents of this paper emphasize some chapters related to the construction of the theoretic model of superluminal particles, tachyon transformation matrix and its application. A particular attention has been devoted to the theoretical model of oscillations of superluminal particles in the domain of the velocities approximate to the speed of light. It has been shown that it is impossible to measure velocities higher that the speed of light in certain circumstances. A particular attention has been devoted to the method of measuring the velocity of neutrinos in contemporary laboratories, as well as to the analysis of the results of the arrival of neutrinos and photons in the laboratories on the Earth emitted during the explosion of Supernova SN1987A.

2. The Transition from Einstein's Physics to the World of Superluminal Physics

The transition from Einstein's physics to the world of superluminal physics is extremely simple. To illustrate that, it is necessary to start from the squared version of Einstein's energy equation

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \tag{1}$$

where

$$\begin{gathered} p = m\gamma v; E = m\gamma c^2 \\ \gamma = \frac{1}{\sqrt{1-(v/c)^2}}; v < c \end{gathered} \tag{2}$$

represents the particle momentum, and m is the rest mass particle, while

c – represents the speed of light

As it is known, this equation can be represented in the form of a right-angled triangle. One of the catheti of such an energy triangle is (pc) and the other is $\left(mc^2\right)$, whilst the energy of the particle E represents the hypotenuse.

To move to the world of tachyons – superluminal particles, only the second cathetus needs to be modified, i.e. to be written in the following form

$$\left(mc^2\right) \to (mcU)$$

where U represents the coordinate speed of the superluminal particle defined by

$$U_i = \frac{dX_i}{dt} > c; i = x, y, z$$

The cathetus (pc) does not change. The other one is modified, but in such a way that the observed mass in the domain $(0,c)$ m_E becomes m

with the transition into the domain $(c, c(1+\delta))$

$$\left(m_E c^2\right) = (mcU) \tag{3}$$

The mass m_E which was primarily the rest mass now becomes the tachyon mass m, mutually connected by the relation

$$m_E = m\frac{U}{c} \tag{4}$$

With such a modification, in a single step, the Einstein's relativistic energy formula provides the energy formula for a superluminal particle

$$\begin{aligned} &E^2 = (pc)^2 + (mcU)^2 \to \\ &E = c\sqrt{p^2 + m^2U^2} \end{aligned} \tag{5}$$

where $m-$ represents the mass of the superluminal particle with minimal speed, which is close to the speed of light but from the top side, and, thereby, it can be explicitly represented by the formula [9]

$$U = c(1+\delta) = c\left(1 + \frac{m^2c^4}{2E^2}\right); \delta << 1 \tag{6}$$

Equation (5) represents a right-angled energy triangle with the hypotenuse E and the catheti (pc) and (mcU).

3. Describing Tachyons Using four Vectors

3.1. Space-like Interval

There is a reference frame where the two events are observed occurring at the same time, but there is not a reference frame in which the two events can occur in the same spatial location. For these space-like event pairs with the positive space-time interval $(S^2 > 0)$, the measuring of space-like separation provides the proper distance

$$\Delta S = \sqrt{S^2} = \sqrt{\Delta r^2 - c^2\Delta t^2} \tag{7}$$

3.2. Time-like Interval

The measuring of a time-like space-time interval is described by the proper time

$$\begin{aligned} \Delta\tau &= \sqrt{\frac{U^2\Delta t^2}{c^2} - \Delta t^2} = \Delta t\sqrt{\frac{U^2}{c^2} - 1} \\ &= \Delta t\frac{1}{\Gamma}; U > c \end{aligned} \tag{8}$$

Thus, it results in

$$\Delta t = \Gamma\Delta\tau \tag{9}$$

And, the definition of the tachyon factor Γ, from here, is

$$\frac{\Delta t}{\Delta\tau} = \Gamma = \frac{1}{\sqrt{\frac{U^2}{c^2} - 1}} \tag{10}$$

3.3. Definition of the Tachyon Velocity

Proper velocity is defined by

$$w = \frac{dX}{d\tau} \tag{11}$$

The tachyon's factor is

$$\Gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{U^2/c^2 - 1}} \tag{12}$$

And the coordinate velocity

$$U = \frac{dX}{dt} \tag{13}$$

that describes an object's rate of motion.

Tachyon four-velocity

The four-coordinate function

$$X^\alpha(\tau) = \begin{pmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \tag{14}$$

defining a tachyon world line, a real function of a variable τ can be simply differentiated in the usual calculus. The tangent will be denoted vector of the tachyon world line $X^\alpha(\tau)$ is a four-dimensional vector called four-velocity,

$$U^\alpha = \frac{dX^\alpha}{d\tau}; \alpha = 0,1,2,3. \tag{15}$$

The relationship between the time t and the coordinate time X^0 is given by

$$X^0 = ct \tag{16}$$

Taking the derivatives with respect to the proper time τ, the U^0 velocity is found

$$U^0 = \frac{d(ct)}{d\tau} = \frac{dt}{d\tau}\frac{d}{dt}(ct) = \Gamma c \tag{17}$$

Using the chain rule for $\alpha = 1,2,3$, it leads to

$$U^{\alpha}=\frac{dX^{\alpha}}{d\tau}=\frac{dt}{d\tau}\frac{d\left(X^{\alpha}\right)}{dt}=\Gamma U_{X^{\alpha}};$$
$$X^{1}=x;X^{2}=y;X^{3}=z \tag{18}$$
$$\alpha=1,2,3$$

Therefore, it results in

$$U^{\alpha}=\begin{pmatrix}U^{0}\\U^{1}\\U^{2}\\U^{3}\end{pmatrix}=\begin{pmatrix}\Gamma c\\\Gamma U_{x}\\\Gamma U_{y}\\\Gamma U_{z}\end{pmatrix}=\begin{pmatrix}\Gamma c\\\Gamma U\end{pmatrix} \tag{19}$$
$$U=\sqrt{U_{x}^{2}+U_{y}^{2}+U_{z}^{2}};\alpha=0,1,2,3.$$

Comment.

In the theory of relativity, the world line is the fabric of the spacetime defined by a similar structural relation

$$x^{\alpha}=\begin{pmatrix}X^{0}\\X^{1}\\X^{2}\\X^{3}\end{pmatrix}=\begin{pmatrix}ct\\X^{1}\\X^{2}\\X^{3}\end{pmatrix} \tag{20}$$

This structure is a foundation of the theory of relativity; the same as the (14) is the foundation of the tachyon theory. In order to differentiate them, they cannot be denoted with the same letters. Proper distance is the same in both structures, but there is the difference in the time component. Therefore, the tachyon structure will be denoted as

$$x^{\alpha}=\begin{pmatrix}X^{0}\\X^{1}\\X^{2}\\X^{3}\end{pmatrix}=\begin{pmatrix}ct_{T}\\X^{1}\\X^{2}\\X^{3}\end{pmatrix}\rightarrow\begin{pmatrix}ct_{T}\\X\end{pmatrix} \tag{21}$$

Thus, in the theory of relativity, the time component is:

$$X^{0}=ct \tag{22}$$

while for a tachyon it is expressed as

$$X^{0}=ct_{T} \tag{23}$$

And, that presents the essential difference between these two theories. This could mean that the value for the measured time in the tachyon laboratory does not match the measured time in the classic laboratory of relativistic physics. And, that means that if classic laboratory measured the time of flight of tachyons, then the obtained result would not describe its real state.

3.4. Tachyon Four-momentum

For a massive particle, the four- momentum is given by the particle's invariant mass m multiplied by the particle's velocity

$$P^{\alpha}=mU^{\alpha}=\begin{pmatrix}P^{0}\\P_{x}\\P_{y}\\P_{z}\end{pmatrix}=\begin{pmatrix}m\Gamma c\\m\Gamma U_{x}\\m\Gamma U_{y}\\m\Gamma U_{z}\end{pmatrix}$$
$$=\begin{pmatrix}m\Gamma c\\m\Gamma U\end{pmatrix} \tag{24}$$
$$p=m\Gamma\sqrt{U_{x}^{2}+U_{y}^{2}+U_{z}^{2}}=m\Gamma U$$

Therefore, this relation provides the momentum of the superluminal particle

$$p=m\Gamma U=\frac{mU}{\sqrt{U^{2}/c^{2}-1}} \tag{25}$$

and the connection with energy

$$E=c\sqrt{p^{2}+m^{2}U^{2}}=\vec{p}\vec{U}$$
$$=m\Gamma U^{2}=\frac{mU^{2}}{\sqrt{U^{2}/c^{2}-1}} \tag{26}$$

On the basis of the equation (26), its other equivalent form is obtained

$$E=\frac{U^{2}}{c}\sqrt{p^{2}-m^{2}c^{2}} \tag{27}$$

4. Klein-Gordon Equation

The proceeding will be carried out as it was made for the special theory of relativity. Thus, it is started with the identity from the tachyon theory describing the energy [10].

$$E=\frac{U^{2}}{c}\sqrt{p^{2}-m^{2}c^{2}}=\sqrt{\frac{U^{4}}{c^{2}}p^{2}-m^{2}U^{4}} \tag{28}$$

Then, just inserting the quantum mechanical operators for momentum and energy yield the equation

$$i\hbar\frac{\partial}{\partial t}=\sqrt{\frac{U^{4}}{c^{2}}\left(-i\hbar\frac{\partial}{\partial x}\right)^{2}-m^{2}U^{4}} \tag{29}$$

However, this equation has no sense, because differential equation cannot be evaluated while under the square root sign. In addition, this equation as it stands, instead begins with the square of the above identity (28), i.e.

$$E^{2}=p^{2}\frac{U^{4}}{c^{2}}-m^{2}U^{4} \tag{30}$$

which when quantized gives

$$\left(i\hbar\frac{\partial}{\partial t}\right)^2\Psi=\frac{U^4}{c^2}\left(-i\hbar\frac{\partial}{\partial x}\right)^2\Psi-m^2U^4\Psi \tag{31}$$

and by simplifying, this equation becomes

$$\frac{\partial^2}{\partial t^2}\Psi-\frac{U^4}{c^2}\frac{\partial^2}{\partial x^2}\Psi-\frac{m^2U^4}{\hbar^2}\Psi=0 \tag{32}$$

Free particle solutions

The Klein-Gordon equation for a free particle can be written as equation (32). We look for plane wave solutions of the form

$$\begin{aligned}\Psi(x,t)&=\exp\left[\frac{i}{\hbar}(\pm px\mp Et)\right]\\&=\exp\left[i(\pm kx\mp\omega t)\right]=\exp(i\phi)\end{aligned} \tag{33}$$

For some constant angular frequency $\omega\in R$ and wave number $k\in R^3$, substitution gives the dispersion relation

$$-\omega^2+\frac{U^4}{c^2}|k|^2=\frac{m^2U^4}{\hbar^2} \tag{34}$$

which represents the equation which satisfies the wave equation (32). Energy and momentum are seen to be proportional the ω and $\vec{k}$

$$\langle\vec{p}\rangle=\langle\Psi|-i\hbar\nabla|\Psi\rangle=\hbar\vec{k} \tag{35}$$

$$\langle E\rangle=\left\langle\Psi\left|i\hbar\frac{\partial}{\partial t}\right|\Psi\right\rangle=\hbar\omega \tag{36}$$

Therefore, dispersion relation is just the energy function of a tachyonic particle:

$$\begin{aligned}\langle E\rangle^2&=\langle p\rangle^2\frac{U^4}{c^2}-m^2U^4\\&=\frac{U^4}{c^2}\left(\langle p\rangle^2-m^2c^2\right)\end{aligned} \tag{37}$$

For the case of the independence of time, i.e., when is

$$\frac{\partial}{\partial t}\Psi=0$$

Klein-Gordon equation reduces to the equation

$$\left(\frac{U^4}{c^2}\frac{\partial^2}{\partial x^2}+\frac{m^2U^4}{\hbar^2}\right)\Psi=0 \tag{38}$$

i.e.

$$\left(\frac{\partial^2}{\partial x^2}+\frac{m^2c^2}{\hbar^2}\right)\Psi=0 \tag{39}$$

Also, and in that case one looks for plane wave solutions of the form

$$\Psi(x)=\exp\left(\frac{i}{\hbar}px\right)=\exp(ikx) \tag{40}$$

where the square of a wave number is

$$k^2=\frac{m^2c^2}{\hbar^2} \tag{41}$$

And the particle momentum is

$$p=\pm\hbar k=\pm mc \tag{42}$$

This momentum represents tachyon when its velocity is [11]

$$U=c(1+n\delta)\approx\frac{c}{\sqrt{2\delta}}=\frac{E}{mc};n\delta>>1 \tag{43}$$

Taking into account (40) and (42), wave function can be written in the form:

$$\Psi(x)=\exp\left(\pm i\frac{mc}{\hbar}x\right)=\exp(\pm ikx) \tag{44}$$

where the sign "plus" is in relation to tachyons, and sign "minus" with antitachyons, and it is apparent that the particle is moving along the x-axis.

5. Tachyonic Transformation Matrix

Apart from the procedure given in [8], a tachyon matrix will be derived here using the principle of correspondence between the theory of relativity and the theory of superluminal particles. Entering a contemporary laboratory from its tachyon world, a superluminal particle retains its energy and impulse. We have seen that, owing to the different domain of velocities $(0,c)$ it would ostensibly change its mass (4). And, if it moves close to the speed of light from the upper side, then its measured mass in a classic laboratory would be ostensibly higher for the amount $1+\delta$. Therefore,

$$m_E=m(1+\delta) \tag{45}$$

To this relation, it is also associated:

The impulse equality

$$m_E\gamma v=m\Gamma U \tag{46}$$

The energy equality

$$m_E\gamma c^2=m\Gamma U^2 \tag{47}$$

On the basis of (45, 46, and 47), the connection among the members of transformation matrices is obtained:

$$\gamma\frac{v}{c}=\gamma\beta=\Gamma;\beta<1 \tag{48}$$

$$\gamma=\Gamma\beta_T;\beta_T>1 \tag{49}$$

On the basis of the relations (48) and (49), the Lorentz matrix transforms into tachyon matrix

$$L=\begin{pmatrix}\gamma & -\gamma\beta\\ -\gamma\beta & \gamma\end{pmatrix}\to T=\begin{pmatrix}\Gamma\beta_T & -\Gamma\\ -\Gamma & \Gamma\beta_T\end{pmatrix}=\begin{pmatrix}\Gamma\frac{U}{c} & -\Gamma\\ -\Gamma & \Gamma\frac{U}{c}\end{pmatrix} \tag{50}$$

5.1. Characteristics of a Tachyon Matrix

A tachyon matrix is an orthogonal matrix with the determinant equaling one:

$$\det T=\det\begin{pmatrix}\Gamma\beta_T & -\Gamma\\ -\Gamma & \Gamma\beta_T\end{pmatrix}=\det T^{-1}=\det\begin{pmatrix}\Gamma\beta_T & \Gamma\\ \Gamma & \Gamma\beta_T\end{pmatrix}=1 \tag{51}$$

In relation to Minkowski space-time metric, the following relations can be shown:

$$\begin{aligned}&TT^{-1}=I\\ &TgT=T^TgT=g\\ &T^{-1}gT^{-1}=\left(T^T\right)^{-1}gT^{-1}=g\\ &gTg=T^{-1}\\ &gT^{-1}g=T\end{aligned} \tag{52}$$

where

$$g=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix} \tag{53}$$

Minkowski space-time metric and

$$\lim_{U\to\infty}T=\lim_{U\to\infty}\begin{pmatrix}\Gamma\frac{U}{c} & -\Gamma\\ -\Gamma & \Gamma\frac{U}{c}\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

5.2. Four-momentum Invariance

For the particle moving along x-direction, starting from (24) and applying the tachyon transformation matrix in the S^1 and S^2 inertial reference frames, the following can be written

$$\begin{pmatrix}m\Gamma_1 c\\ m\Gamma_1 U_1\end{pmatrix}=\begin{pmatrix}\Gamma\frac{U}{c} & -\Gamma\\ -\Gamma & \Gamma\frac{U}{c}\end{pmatrix}\begin{pmatrix}m\Gamma_2 c\\ m\Gamma_2 U_2\end{pmatrix} \tag{54}$$

From this, one gets

$$\begin{aligned}&m\Gamma_1 c=\Gamma\frac{U}{c}m\Gamma_2 c-\Gamma m\Gamma_2 U_2;\\ &m\Gamma_1 U_1=\Gamma\frac{U}{c}m\Gamma_2 U_2-\Gamma m\Gamma_2 c;\\ &\Gamma=\left(U^2/c^2-1\right)^{-1/2}\end{aligned} \tag{55}$$

$$\Gamma_1=\left(U_1^2/c^2-1\right)^{-1/2};\Gamma_2=\left(U_2^2/c^2-1\right)^{-1/2}$$

Four-momentum invariant quantity is obtained with the following process:

$$\begin{aligned}&m^2\Gamma_1^2c^2-m^2\Gamma_1^2U_1^2=\left(m\Gamma\Gamma_2U-m\Gamma\Gamma_2U_2\right)^2\\ &-\left(m\Gamma\Gamma_2\frac{U}{c}U_2-m\Gamma\Gamma_2c\right)^2=-m^2c^2\end{aligned} \tag{56}$$

Also, using Minkowski space-time metric, one gets the same invariant quantity

$$\begin{aligned}&\begin{pmatrix}m\Gamma c\\ m\Gamma U_x\\ m\Gamma U_y\\ m\Gamma U_z\end{pmatrix}^+\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}m\Gamma c\\ m\Gamma U_x\\ m\Gamma U_y\\ m\Gamma U_z\end{pmatrix}\\ &=m^2\Gamma^2c^2-m^2\Gamma^2U^2=m^2\Gamma^2c^2\left(1-\frac{U^2}{c^2}\right)\\ &=-m^2c^2\end{aligned} \tag{57}$$

5.3. Spacetime Fabric

For the tachyon energy (26), spacetime fabric is defined

$$\begin{pmatrix}ct_T\\ X\end{pmatrix} \tag{58}$$

where ct_T represents a time component and

$$X=\begin{pmatrix}x\\y\\z\end{pmatrix}$$

is a space coordinate. In the theory of relativity, spacetime fabric is defined with the matrix

$$\begin{pmatrix} ct_E \\ X \end{pmatrix} \tag{59}$$

where ct_E is a time component. Thus, as it can be seen, the essential difference between (58) and (59) lies in different time components. These components depend on the laboratory in which the measurements are conducted. In a tachyon laboratory, the measured time is t_T, whereas in a classic contemporary laboratory, it is t_E and it is usually labelled as t. The difference between these two times is particularly emphasized, because if there was no difference in time component, there would be no difference between the theory of relativity and the tachyon theory that is proposed here.

Thus, officially, as far as it is known in contemporary physics, there is neither the physics of faster-than-light particles with real mass nor there is a laboratory for their potential detection. If, by chance, a contact with such particles existed in a classic laboratory, without knowing they were superluminal, the misinterpretation of some of their physical features could occur. Such interpretation would happen because the two times in (58) and (59) are connected by the relation [11]

$$t_T = \frac{t_E}{(1+\delta)^2}; \delta << 1 \tag{60}$$

It is useful to remember where this relation originates from. The travelled distance X is defined as proper distance for both velocity domains and it equals the product of proper velocities and proper times:

$$\begin{aligned} X &= w_E \tau_E = (\gamma v_E)\left(\frac{1}{\gamma} t_E\right) = v_E t_E \\ &= w_T \tau_T = (\Gamma U)\left(\frac{1}{\Gamma} t_T\right) = U t_T \end{aligned} \tag{61}$$

If one switches to the domain of velocities below the light barrier, which is the domain that belongs to classic laboratories that observe the particles that subject to the laws of classic and relativistic physics, then it is inevitable to have the change of the spacetime structure, as shown below with an arrow

$$\begin{pmatrix} ct_T \\ X \end{pmatrix} \to \begin{pmatrix} ct_E \\ X \end{pmatrix}$$

The travelled distance X remains unchanged, while there is a change in the time component: $ct_T \to ct_E$. These relations become more apparent if transformed in the travelled distance X:

$$\begin{aligned} ct_T &= \frac{c(1+\delta)}{1+\delta} t_T = \frac{U t_T}{1+\delta} \approx X - X \bullet \delta \to \\ ct_E &= t_E \frac{c}{1+\delta}(1+\delta) = t_E v_E (1+\delta) \\ &= X + X \bullet \delta \end{aligned} \tag{62}$$

Thus, the change of the time component, due to the difference in the spacetime structure, leads to the change in the result of the measured time. If that change is particularly highlighted

$$X - X \bullet \delta \to X + X \bullet \delta \tag{63}$$

it could be seen that the time component ct_T from the domain of velocity $(c,(1+\delta)c)$ ostensibly dilates with the transfer in the domain of velocity of the relativistic physics $(0,c)$. Therefore, transferring from the domain of superluminal velocity over the light barrier to the domain of relativistic physics, measuring time in a laboratory, the conclusion is reached that the travelling time of a superluminal particle over the distance X is larger than the light photon. That at the same time means that it is impossible to measure the velocity of superluminal particles in classic laboratories. In this theory, a tachyon has been considered as a particle that possesses real mass, but that is without rest mass. Thus, for tachyons as superluminal particles, there are no rest frames because their velocities are restricted to below the speed of light. However, the velocities are not restricted to above it and, therefore, the limit of infinite velocities may always be considered. It can be stated that the difference in the measured time can occur due to the manner of measuring the time of flight of particles over a distance. Then, the time of travelling of particles is monitored over reference systems that obligatorily require the usage of Lorentz transformation matrix. However, if the laboratory is in contact with particles of unknown origin and if they are faster than light, then the usage of Lorentz transformation matrix can inevitably lead to wrong results. Namely, the members of Lorentz matrix for speeds higher than the speed of light become imaginary numbers as well as the travelled distances. Thus, one can say that it is impossible to measure the velocity of such particles by monitoring the time of flight over a distance. If it was monitored without knowing it was a tachyon particle, using the Lorentz matrix, a paradoxical result would be obtained for the time of flight t_E (62). The reason for that being a paradoxical value shall be discussed in following sections.

Comment. The paradoxical results occur because there is no rest reference frame attached to the tachyon world lines and, in that sense, special relativity does not really treat tachyons on the same footing as slower-than-light particles.

6. Introducing Tachyon's Transformation Matrix

Let S and S^1 be reference frames along coordinate systems

$\left(ct_T \quad x, \quad y, \quad z\right)$ and $\left(ct_T^1 \quad x_1, \quad y_1, \quad z_1\right)$ to be defined. Let their corresponding axes be aligned with the x and x_1 axes along the line of relative motion so that S^1 has velocity U along x direction in reference frame S.

In addition, let the origins of coordinates and time be chosen so that the origins of the two reference frames coincide at $t_T = t_T^1 = 0$. Thus, if an event has coordinates $\left(ct_T \quad x, \quad y, \quad z\right)$ in S, then the coordinates in S^1 are given by

$$\begin{pmatrix} ct_T^1 \\ x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T & -\Gamma & 0 & 0 \\ -\Gamma & \Gamma\beta_T & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct_T \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T ct_T - \Gamma x & 0 & 0 & 0 \\ -\Gamma ct_T + \Gamma\beta_T x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \end{pmatrix} \tag{64}$$

From here, one has

$$\begin{aligned} ct_T^1 &= \Gamma\beta_T ct_T - \Gamma x = \Gamma\left(\beta_T ct_T - x\right) \\ x_1 &= \Gamma\beta_T x - \Gamma ct_T = \Gamma\left(\beta_T x - ct_T\right) \\ y_1 &= y \\ z_1 &= z \end{aligned} \tag{65}$$

where

$$\Gamma = \frac{1}{\sqrt{\beta_T^2 - 1}}; \beta_t = \frac{U}{c} > 1$$

By solving for $\left(ct_T \quad x, \quad y, \quad z\right)$ in terms of $\left(ct_T^1 \quad x_1, \quad y_1, \quad z_1\right)$, the inverse tachyon transformation is used:

$$\begin{pmatrix} ct_T \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T & \Gamma & 0 & 0 \\ \Gamma & \Gamma\beta_T & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct_T^1 \\ x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T ct_T^1 + \Gamma x_1 & 0 & 0 & 0 \\ \Gamma ct_T^1 + \Gamma\beta_T x_1 & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \end{pmatrix} \tag{66}$$

From here, it follows:

$$\begin{aligned} ct_T &= \Gamma\beta_T ct_T^1 + \Gamma x_1 = \Gamma\left(\beta_T t_T^1 + x_1\right) \\ x &= \Gamma\beta_T x_1 + \Gamma ct_T^1 = \Gamma\left(\beta_T x_1 + ct_T^1\right) \\ y &= y_1 \\ z &= z_1 \end{aligned} \tag{67}$$

6.1. Time Dilatation

Two events separated by a time interval Δt_T on the same space location in the reference system S are observed. The coordinate beginning is labelled with $X = 0$ and for time from $t_T = 0$ to Δt_T the equation (66) is applied to the two events. The equation can be extracted

$$t_T^1 = \Gamma\beta_T t_T - \Gamma\frac{X}{c} \tag{68}$$

The first event occurs at $t_T = 0$ and the other at Δt_T^1 which needs to be determined. In the equation (68) we put $t_T = 0$ therefore, one has

$$t_T^1 = -\Gamma\frac{X}{c} \tag{69}$$

For the other event in the time interval Δt_T, the equation (68) is written in the form

$$t_{T1}^1 = \Gamma\beta_T\Delta t_T - \Gamma\frac{X}{c} \tag{70}$$

Subtracting left and right sides of the equations (69) and (70), the time dilatation is obtained

$$t_{T1}^1 - t_T^1 = \Delta t_T^1 = \Gamma\beta_T\Delta t_T \tag{71}$$

6.2. Length Contraction

In order to determine the length contraction, there will be no usage of two events, but of two world lines. Those are the world lines of two ends of an object fixed in the reference system S in the $\boldsymbol{x}$–axis direction. The beginning is set at one of those world lines and the other end is in the position $X = L$, where L is rest length. Let us put $t_T^1 = 0$ and observe world lines in the reference system S^1. At that moment, the world line passes through the beginning of the reference system S^1 when $X^1 = 0$ and in the system of equations of the other world line

$$t_T^1 = \Gamma\beta t_T - \Gamma\frac{X}{c} \tag{72}$$

$$X^1 = -\Gamma ct_T + \Gamma\beta X \tag{73}$$

From the first equation for $t_T^1 = 0$, one finds

$$t_T = \frac{X}{c\beta}; \beta > 1 \tag{74}$$

by replacing it in (73), the length contraction is found

$$X^1 = -\Gamma c \frac{X}{c\beta} + \Gamma\beta X = \Gamma X \frac{\beta^2 - 1}{\beta} = \frac{X}{\Gamma\beta} \tag{75}$$

The product of time dilation and length contraction in any inertial reference system

$$X^1 \Delta t_T^1 = \frac{X}{\Gamma\beta}(\Gamma\beta)\Delta t = X\Delta t_T \tag{76}$$

is mutually equal.

6.3. Velocity

The transformation between two reference systems of energy and impulse – momentum is written in the following manner:

$$\begin{pmatrix} E_2^1 \\ p_2^1 c \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T & -\Gamma \\ -\Gamma & \Gamma\beta_T \end{pmatrix} \begin{pmatrix} E_2 \\ p_2 c \end{pmatrix} \tag{77}$$

Putting that $E_2^1 = p_2^1 U_2^1, E_2 = p_2 U_2$, it will be:

$$\begin{gathered} \left(\frac{E_2^1}{p_2^1 c}\right)^2 = \left(\frac{U_2^1}{c}\right)^2 = \frac{(\beta_Y U_2 - c)^2}{(\beta_Y c - U_2)^2} > 1 \rightarrow \\ (\beta_Y U_2)^2 + c^2 > (\beta_Y c)^2 + U_2^2 \\ \beta_T > 1; U_2 > c \end{gathered} \tag{78}$$

If it is applied in inverse tachyon transformation matrix

$$\begin{pmatrix} E_2 \\ p_2 c \end{pmatrix} = \begin{pmatrix} \Gamma\beta_T & \Gamma \\ \Gamma & \Gamma\beta_T \end{pmatrix} \begin{pmatrix} E_2^1 \\ p_2^1 c \end{pmatrix} \tag{79}$$

one will get

$$\begin{gathered} \left(\frac{U_2}{c}\right) = \frac{(\beta_Y U_2^1 + c)}{(\beta_Y c + U_2^1)} > 1 \rightarrow \\ (\beta_Y U_2^1) + c > (\beta_Y c) + U_2^1 \\ \beta_T > 1; U_2^1 > c; U_2 > c \end{gathered} \tag{80}$$

6.4. Rapidity

In a space-time diagram the velocity parameter

$$\begin{aligned} \beta_T &= \frac{U}{c} = \coth\Phi = \frac{\cosh\Phi}{\sinh\Phi} \\ &= \frac{e^{\Phi} + e^{-\Phi}}{e^{\Phi} - e^{-\Phi}}; e^{\Phi} = \sqrt{\frac{\beta_T + 1}{\beta_T - 1}} \end{aligned} \tag{81}$$

is the analog of slope and the quantity Φ represents the tachyon rapidity. In terms of rapidity one obtains:

$$\begin{aligned} \Gamma &= \frac{1}{\sqrt{\beta_T^2 - 1}} = \frac{1}{\sqrt{\coth^2\Phi - 1}} = \sinh\Phi \\ \Gamma\beta_T &= \sinh\Phi\coth\Phi = \cosh\Phi \end{aligned} \tag{82}$$

Therefore, the tachyon transformation matrix is

$$T = \begin{pmatrix} \cosh\Phi & -\sinh\Phi & 0 & 0 \\ -\sinh\Phi & \cosh\Phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{83}$$

And, its inverse matrix is

$$T^{-1} = \begin{pmatrix} \cosh\Phi & \sinh\Phi & 0 & 0 \\ \sinh\Phi & \cosh\Phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{84}$$

6.5. The Addition Law - Composition Law of Velocity

Let the addition of velocity be defined as

$$\coth\Phi = \frac{U}{c} \tag{85}$$

and

$$\coth\Phi_1 = \frac{U_1}{c}; \coth\Phi_2 = \frac{U_2}{c} \tag{86}$$

where, if one puts $\Phi = \Phi_1 + \Phi_2$, one gets the relation for composite velocity

$$\begin{aligned} \coth\Phi &= \frac{U}{c} = \coth(\Phi_1 + \Phi_2) \\ &= \frac{\cosh(\Phi_1 + \Phi_2)}{\sinh(\Phi_1 + \Phi_2)} = \frac{\coth\Phi_1 \coth\Phi_2 + 1}{\coth\Phi_1 + \coth\Phi_2} \end{aligned} \tag{87}$$

From here, one finds the resulting or composite velocity

$$U = \frac{U_1 U_2 + c^2}{U_1 + U_2} \tag{88}$$

Composite velocity can be determined in two more ways. One way is by using the Tachyon transformation matrix

$$\begin{pmatrix} ct \\ X \end{pmatrix} = \begin{pmatrix} \Gamma\beta & \Gamma \\ \Gamma & \Gamma\beta \end{pmatrix} \begin{pmatrix} ct^1 \\ X^1 \end{pmatrix} \tag{89}$$

From here, we one finds:

$$t = \Gamma\beta t^1 + \Gamma\frac{X^1}{c}$$
$$X = \Gamma ct^1 + \Gamma\beta X^1 \tag{90}$$

By differentiating the left and right side, it follows

$$\frac{dX}{dt} = \frac{dX}{dt^1}\frac{dt^1}{dt} = \frac{c+\beta\frac{dX^1}{dt^1}}{\beta+\frac{1}{c}\frac{dX^1}{dt^1}} = \frac{c^2+UU_1}{U+U_1}$$
$$\beta = \frac{U}{c} > 1 \tag{91}$$

Using the inverse matrix:

$$\begin{pmatrix} ct^1 \\ X^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix}\begin{pmatrix} ct \\ X \end{pmatrix} \tag{92}$$

It is found

$$t^1 = \Gamma\beta t - \Gamma\frac{X}{c}$$
$$X^1 = \Gamma ct - \Gamma\beta X$$

And from here

$$\frac{dX^1}{dt^1} = \frac{dX^1}{dt}\frac{dt}{dt^1} = \frac{c-\beta\frac{dX}{dt}}{\beta-\frac{1}{c}\frac{dX}{dt}} = \frac{c^2-U\frac{dX}{dt}}{U-\frac{dX}{dt}}$$
$$\beta = \frac{U}{c} > 1$$

Another way for determining the composite velocity (88) can be expressed through equivalent algebraic form

$$\frac{U-c}{U+c} = \left(\frac{U_1-c}{U_1+c}\right)\left(\frac{U_2-c}{U_2+c}\right) \tag{93}$$

solving of which by U , one gets (88).

7. Oscillations of Superluminal Particles

First of all, it is significant to show that the phase of the wave obtained on the basis of Klein-Gordon equation (33) is an invariant quantity. The following transformation formulas present a starting point:

$$\begin{pmatrix} E^1 \\ p^1c \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix}\begin{pmatrix} E \\ pc \end{pmatrix} \tag{94}$$

$$\begin{pmatrix} ct^1 \\ x^1 \end{pmatrix} = \begin{pmatrix} \Gamma\beta & -\Gamma \\ -\Gamma & \Gamma\beta \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix} \tag{95}$$

and the phase of the wave

$$\phi = \frac{1}{\hbar}\left(px - Et\right) \tag{96}$$

On the basis of the equations (94) and (95), one finds:

$$E^1 = \Gamma\beta E - \Gamma pc$$
$$t^1 = \Gamma\beta t - \Gamma\frac{x}{c} \tag{97}$$

Then, the product of the left and right sides of the equations (97) is formed:

$$E^1t^1 = \left(\Gamma\beta E - \Gamma pc\right)\left(\Gamma\beta t - \Gamma\frac{x}{c}\right)$$
$$= \Gamma^2\beta^2 Et - \Gamma^2\beta E\frac{x}{c} - \Gamma^2\beta pct + \Gamma^2 px \tag{98}$$

The following equations are written:

$$p^1 = -\Gamma\frac{E}{c} + \Gamma\beta p$$
$$x^1 = -\Gamma ct + \Gamma\beta x \tag{99}$$

In the same manner, one forms the product of the left and right sides of the equations (99):

$$p^1x^1 = \left(-\Gamma\frac{E}{c} + \Gamma\beta p\right)\left(-\Gamma ct + \Gamma\beta x\right)$$
$$= \Gamma^2 Et - \Gamma^2\beta\frac{E}{c}x - \Gamma^2\beta ct + \Gamma^2\beta^2 px \tag{100}$$

And then, the difference is made between (98) and (100)

$$p^1x^1 - E^1t^1 = px - Et \tag{101}$$

This results means that the phase of the wave (96) remains the same regardless of the reference system in which it is observed. Thus, according to that, it represents the tachyon invariant quantity and that result enables the introduction of the assumption on mutual oscillation between different types of tachyons, the same as that physical phenomenon is present between neutrinos.

In the neutrino physics, experiments have shown that the flavor states $\nu_\alpha, \alpha = e, \mu, \tau$ do not coincide with the mass eigenstates $\nu_i, i = 1,2,3$.

The flavor states are combinations of the mass eigenstates

$$\nu_\alpha = U_{\alpha i}\nu_i$$

and vice versa. Where the mixing parameter $U_{\alpha i}$ forms the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix, U_{PMNSi} . The definition of neutrino oscillations was created on the bases of observations of neutrino oscillations in various neutrino experiments and it has shown that there is a mismatch between the flavor and mass eigenstates of neutrinos.

Let it be assumed that there are two flavor states for tachyons $|t_\alpha\rangle$ and $|t_\beta\rangle$. Two types of superluminal particles will be considered and oscillations will be investigated $(t_\alpha \to t_\beta \to t_\alpha)$. The assumption is introduced stating that two mass eigenstates $|t_1\rangle$ and $|t_2\rangle$ that compose the initially produced flavor state $|t_\alpha\rangle$ have the same energy, and that the spatial propagation of these mass eigenstates can be described by the following phase factor:

$$\begin{aligned} |t_1\rangle &= \exp\left[\frac{i}{\hbar}(p_1 x - E_1 t)\right]|0\rangle \\ &= \exp(i\phi_1)|0\rangle \end{aligned} \tag{102}$$

$$\begin{aligned} |t_2\rangle &= \exp\left[\frac{i}{\hbar}(p_2 x - E_2 t)\right]|0\rangle \\ &= \exp(i\phi_2)|0\rangle \end{aligned} \tag{103}$$

Comment. If just

ϕ_1 (102) and ϕ_2 (103) were observed in isolation, one would get:

$$\begin{aligned} \phi_1 &= \frac{1}{\hbar}(p_1 x - E_1 t) \\ &= \frac{1}{\hbar}(p_1 U_1 t - p_1 U_1 t) = 0 \end{aligned} \tag{104}$$

$$\begin{aligned} \phi_2 &= \frac{1}{\hbar}(p_2 x - E_2 t) \\ &= \frac{1}{\hbar}(p_2 U_2 t - p_2 U_2 t) = 0 \end{aligned} \tag{105}$$

The results that have no physical sense are obtained.

That means that mass eigenstates $|t_1\rangle$ and mass eigenstate $|t_2\rangle$ cannot exist in isolation, on their own. The real and final value of the phase exists only if mass eigenstates are mutually connected

$$\begin{aligned} \phi_1 &= \frac{1}{\hbar}(p_1 x - E_1 t) \\ : &= \frac{1}{\hbar}\left(p_1 \frac{U_1 + U_2}{2} t - p_1 U_1 t\right) \\ &= \frac{1}{\hbar}\left(p_1 \frac{U_2}{2} - \frac{p_1 U_1}{2}\right) t \neq 0 \\ E_1 &= p_1 U_1 \end{aligned} \tag{106}$$

$$\begin{aligned} \phi_2 &= \frac{1}{\hbar}(p_2 x - E_2 t) \\ &= \frac{1}{\hbar}\left(p_2 \frac{U_1}{2} - \frac{p_2 U_2}{2}\right) t \neq 0; \\ E_2 &= p_2 U_2 \end{aligned} \tag{107}$$

Thus, tachyons either do not exist, which is shown by the results of the relations (105) and (104), or they exist if they unite, as shown by the relations (106) and (107). These tachyon features could be deciding in defining the nature of particles that participate in oscillations. Namely, according to the theory on neutrino oscillations, flavor states are combinations of the mass eigenstates. In the formulas (106) and (107), it can be seen that the phases depend both on the speed of one mass eigenstate $|\nu_1\rangle$ and on the speed of the other mass eigenstate $|\nu_2\rangle$, which indicates their mutual correlation.

7.1. Definition of Flavor Oscillations

Let it be supposed that one starts with a beam of tachyons with some amount of mass eigenstates $|t_\alpha\rangle$ and $|t_\beta\rangle$ at the space-time point (0,0) and that the beam is directed along the x-axis. Let tachyons propagate in a vacuum to a detector at some distance L from the generation point. Using the plane-wave solution to the Klein-Gordon equation (33), the mass states at some space-time point (ct, x) can be written as

$$\begin{pmatrix} |t_1(x,t)\rangle \\ |t_2(x,t)\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\phi_1} & 0 \\ 0 & e^{-i\phi_2} \end{pmatrix} \begin{pmatrix} |t_1(0,0)\rangle \\ |t_2(0,0)\rangle \end{pmatrix} \tag{108}$$

And, the flavor states at some space-time point (ct, x) can be written as

$$\begin{aligned} \begin{pmatrix} |t_\alpha(x,t)\rangle \\ |t_\beta(x,t)\rangle \end{pmatrix} &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |t_1(x,t)\rangle \\ |t_2(x,t)\rangle \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\phi_1} & 0 \\ 0 & e^{-i\phi_2} \end{pmatrix} \\ &\quad \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |t_\alpha(0,0)\rangle \\ |t_\beta(0,0)\rangle \end{pmatrix} \end{aligned} \tag{109}$$

On the basis of the calculated values for phases (104) and (105) of mutually independent tachyons, inserting them in the relations (108), one finds:

$$\begin{aligned} \begin{pmatrix} |t_1(x,t)\rangle \\ |t_2(x,t)\rangle \end{pmatrix} &= \begin{pmatrix} e^{-i0} & 0 \\ 0 & e^{-i0} \end{pmatrix} \begin{pmatrix} |t_1(0,0)\rangle \\ |t_2(0,0)\rangle \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} |t_1(0,0)\rangle \\ |t_2(0,0)\rangle \end{pmatrix} = \begin{pmatrix} |t_1(0,0)\rangle \\ |t_2(0,0)\rangle \end{pmatrix} \end{aligned} \tag{110}$$

From here, one finds:

$$|t_1(ct,x)\rangle = |t_1(0,0)\rangle \tag{111}$$

$$|t_2(ct,x)\rangle = |t_2(0,0)\rangle \tag{112}$$

And

$$\begin{aligned}
&\begin{pmatrix} |t_\alpha(x,t)\rangle \\ |t_\beta(x,t)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i0} & 0 \\ 0 & e^{-i0} \end{pmatrix} \\
&\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} t_\partial(0,0) \\ t_\beta(0,0) \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} t_\partial(0,0) \\ t_\beta(0,0) \end{pmatrix} = \begin{pmatrix} |t_\alpha(0,0)\rangle \\ |t_\beta(0,0)\rangle \end{pmatrix}
\end{aligned} \tag{113}$$

From here, it follows:

$$|t_\alpha(ct,x)\rangle = |t_\alpha(0,0)\rangle \tag{114}$$

$$|t_\beta(ct,x)\rangle = |t_\beta(0,0)\rangle \tag{115}$$

For mutually independent tachyons, the calculated values (111, 112, 114, 115) mean that there are no changes in time during the propagation of tachyons from the source to the detector, nor mass eigenstates $|t_{1,2}(ct,x)\rangle$, nor flavor states $|t_{\alpha,\beta}(ct,x)\rangle$. Practically, that would mean that such particles, which are mutually independent, could not exist in nature. And, if there was a question under what conditions such superluminal particles could be found in nature, then, the only answer could be in the calculated, mutually dependent phases of theirs, (106) and (107). Therefore, a significant conclusion that could be made out of this deliberation is:

1. Tachyons could not exist as independent particles in nature.
2. And, if they did exist, then they would appear in the state in which they are mutually connected. In other words: the flavor state of tachyons would be shown through a unitary matrix as a combination of mass eigenstates. And vice versa: each mass state would present a combination of flavor states. These tachyon characteristics would meet the definition for the existence of the mismatch between flavor states and mass states. According to that, the particles with those characteristics would mutually oscillate.

Comment. In the relation (106), it can be seen that the phase ϕ_1 of the mass eigenstate $|t_1\rangle$ depends also on the velocity U_2, and in the relation (107), it can be seen that the phase ϕ_2 of the mass eigenstate $|t_2\rangle$ depends also on the velocity U_1. Therefore, these combinations of mass eigenstates $|t_1\rangle$ and $|t_2\rangle$ are in accordance with the definition of two flavor oscillations.

In the first step for finding the formula for the oscillation length, it is necessary to form the difference between (106) and (107), under the condition that the energies are mutually equal, i.e. $E_1 = E_2$ (Equal energy assumption)

$$\begin{aligned}
\phi_1 - \phi_2 &= \frac{1}{\hbar}\left(p_1 \frac{U_2}{2}\frac{2x}{U_1+U_2} - p_2\frac{U_1}{2}\frac{2x}{U_1+U_2} \right) \\
&= \frac{1}{\hbar}\left(\frac{p_1U_2 - p_2U_1 + p_1U_1 - p_2U_2}{U_1+U_2} \right)x \\
&= \frac{1}{\hbar}(p_1 - p_2)x; p_1U_1 = p_2U_2
\end{aligned} \tag{116}$$

where, of course, due to the mutual feedback between mass eigenstate $|t_1\rangle$ and mass eigenstate $|t_2\rangle$, the distance travelled is at every point equal to the product of average velocity and time.

$$x = \frac{U_1+U_2}{2}t = \bar{U}t \tag{117}$$

Comment. The difference of the phases (116) can be obtained by not introducing mean values, but by simply putting that the times are equal, i.e. $t_1 = t_2 = t$.

7.2. Subluminal Neutrino Phases

Let the phases of mass eigenstates (102) and (103) in spacetime $(ct_E \quad X)$ of relativistic physics be observed

$$\begin{aligned}
\phi_1 &= \frac{1}{\hbar}(p_1x - E_1t_T) \\
&= \frac{1}{\hbar}\left(p_1v_1t_E - \frac{p_1c^2}{v_1}\frac{t_E}{(1+\delta_1)^2} \right) = 0
\end{aligned} \tag{118}$$

$$\begin{aligned}
\phi_2 &= \frac{1}{\hbar}(p_2x - E_2t_T) \\
&= \frac{1}{\hbar}\left(p_2v_2t_E - \frac{p_2c^2}{v_2}\frac{t_E}{(1+\delta_2)^2} \right) = 0
\end{aligned} \tag{119}$$

However, these are just mapped zero phases (104) and (105) of tachyons from the domain of speed $(c,(1+\delta)c)$ into the relativistic domain $(0,c)$.

Comment. The phases (105) and (104) are mapped from the tachyon spacetime into the relativistic spacetime in (118) and (119). Zero values are obtained as (105) and (104). However, if neutrinos are considered to be subluminal particles, then there are the following expressions for the phases:

$$\begin{aligned}
\phi_1 &= \frac{1}{\hbar}(p_1x - E_1t_1) = \frac{1}{\hbar}\left(p_1x - E_1\frac{x}{v_1} \right) \\
&= \frac{1}{\hbar}p_1x\left(1 - \frac{E_1}{p_1v_1} \right) = \frac{1}{\hbar}p_1x\left(1 - \frac{c^2}{v_1^2} \right) \\
&= \frac{1}{\hbar}p_1x(1-1-2\delta_1) = -\frac{x}{\hbar}p_1 2\delta_1
\end{aligned} \tag{120}$$

$$\phi_2=\frac{1}{\hbar}\left(p_2x-E_2t_2\right)=\frac{1}{\hbar}\left(p_2x-E_2\frac{x}{v_2}\right)$$
$$=\frac{1}{\hbar}p_2x\left(1-\frac{E_2}{p_2v_2}\right)=\frac{1}{\hbar}p_2x\left(1-\frac{c^2}{v_2^2}\right) \tag{121}$$
$$=\frac{x}{\hbar}p_2\left(1-1-2\delta_2\right)=-\frac{x}{\hbar}p_22\delta_2$$

For further calculation, the approximation relations of relativistic physics will be used:

$$p_1=\frac{E_1}{c}\left(1-\frac{m_1^2c^4}{2E_1^2}\right)=\frac{E}{c}\left(1-\delta_1\right) \tag{122}$$

$$p_2=\frac{E_2}{c}\left(1-\frac{m_2^2c^4}{2E_2^2}\right)=\frac{E}{c}\left(1-\delta_2\right) \tag{123}$$

The results of (120) and (121) are final values for the phases and they show the mutual independence between mass eigenstate $|\nu_1\rangle$ and mass eigenstate $|\nu_2\rangle$. That means that mass eigenstates (120) and (121) can exist independently, thus, owing to that, a mismatch between flavor states and mass eigenstates could not be created.

However, if a difference is not made between the times, as is usually the case in the neutrino physics, $t_1=t_2=t$ there will be:

$$\phi_1=\frac{1}{\hbar}\left(p_1x-E_1t\right)\neq\frac{1}{\hbar}\left(p_1x-E_1t_1\right)$$

$$\phi_2=\frac{1}{\hbar}\left(p_2x-E_2t\right)\neq\frac{1}{\hbar}\left(p_2x-E_2t_2\right)$$

Comment. The phases for mass eigenstates (102) and (103) cannot independently exist because of (104) and (105), but they must unite and mutually combine, as is given in the expressions (106), (107) and (116).

That mutual dependence of theirs actually represents the necessary combination for creating the flavor state. And that is essentially in agreement with the definition for the flavor state, which is experimentally confirmed and which states: Observations of neutrino oscillations in various neutrino experiments have shown that there is a mismatch between the flavor and mass eigenstates of neutrinos. That definition for the flavor state is crucial for the recognition of the oscillation phenomenon. Simply put: if there is a mismatch between the flavor state and mass eigenstates, then those particles mutually oscillate.

As for the phases (120) and (121), they are equal to zero just at the initial moment for $t=0$ and, with every further flow of time, they are different from zero. They are mutually independent, which means that a certain flavor state could be bound to either one mass eigenstate $|\nu_1\rangle$ or the other mass eigenstate $|\nu_2\rangle$. These possibilities violate the rule of the flavor state definition, stating that it is defined exclusively as the combination of these mass eigenstates. The violation of that rule opposes the experiments that have shown that there is a mismatch between the flavor and mass eigenstates of neutrinos. According to that, by bonding exclusively with either mass eigenstate $|\nu_1\rangle$ or with mass eigenstate $|\nu_2\rangle$, there would be no mismatch between the flavor and mass eigenstates of neutrinos, which opposes the realization of conditions for the occurrence of oscillations. Thus, solely on the basis of the aforesaid, it could be concluded that subliminal neutrinos could not participate in the oscillation process.

Everything said can be described by transformations that essentially present the definition for the oscillation between particles.

The first transformation

$$\begin{pmatrix}|\nu_1(x,t)\rangle\\|\nu_2(x,t)\rangle\end{pmatrix}=\begin{pmatrix}e^{-i\phi_1}&0\\0&e^{-i\phi_2}\end{pmatrix}\begin{pmatrix}|\nu_1(0,0)\rangle\\|\nu_2(0,0)\rangle\end{pmatrix} \tag{124}$$

connects mass eigenstates

$$|\nu_1(x,t)\rangle=e^{-i\phi_1}|\nu_1(0,0)\rangle \tag{125}$$

$$|\nu_2(x,t)\rangle=e^{-i\phi_2}|\nu_2(0,0)\rangle \tag{126}$$

The next transformation connects the flavor state that is by definition the combination of mass eigenstates over mixing angle θ

$$\begin{pmatrix}|\nu_\alpha(x,t)\rangle\\|\nu_\beta(x,t)\rangle\end{pmatrix}=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}|\nu_1(x,t)\rangle\\|\nu_2(x,t)\rangle\end{pmatrix}$$
$$=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}e^{-i\phi_1}&0\\0&e^{-i\phi_2}\end{pmatrix} \tag{127}$$
$$\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}|\nu_\alpha(x,t)\rangle\\|\nu_\beta(x,t)\rangle\end{pmatrix}$$

However, one can see that the phases (120) and (121) are mutually independent, which means that there is no mixing between flavor states and mass states, therefore it can be put in (127) $\theta=0$. Then, (127) becomes:

$$\begin{pmatrix}|\nu_\alpha(x,t)\rangle\\|\nu_\beta(x,t)\rangle\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}|\nu_1(x,t)\rangle\\|\nu_2(x,t)\rangle\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}e^{-i\phi_1}&0\\0&e^{-i\phi_2}\end{pmatrix} \tag{128}$$
$$\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}|\nu_\alpha(0,0)\rangle\\|\nu_\beta(0,0)\rangle\end{pmatrix}$$

On the basis of the relation (128), it can be written:

$$\left|\nu_\alpha(x,t)\right\rangle = \left|\nu_1(x,t)\right\rangle = e^{-i\phi_1}\left|\nu_1(0,0)\right\rangle \tag{129}$$

$$\left|\nu_\beta(x,t)\right\rangle = \left|\nu_2(x,t)\right\rangle = e^{-i\phi_2}\left|\nu_2(0,0)\right\rangle \tag{130}$$

$$\left|\nu_\alpha(x,t)\right\rangle = e^{-i\phi_1}\left|\nu_\alpha(0,0)\right\rangle \tag{131}$$

$$\left|\nu_\beta(x,t)\right\rangle = e^{-i\phi_2}\left|\nu_\beta(0,0)\right\rangle \tag{132}$$

Comparing (129) with (131) and (130) with (132), one finds:

$$\left|\nu_1(0,0)\right\rangle = \left|\nu_\alpha(0,0)\right\rangle \tag{133}$$

$$\left|\nu_2(0,0)\right\rangle = \left|\nu_\beta(0,0)\right\rangle \tag{134}$$

Mutual independence of the phases (120) and (121) leads to the results (133) and (134) that do not meet the conditions for the occurrence of a mismatch between flavor states and mass states. That means that subluminal neutrinos could not exist in nature.

7.3. Determining the Oscillation Length

Superluminal particle of mass eigenstate $\left|t_1\right\rangle$ with momentum

$$p_1 = m_1\Gamma_1 U_1 \tag{135}$$

is the energy eigenstate with eigenvalue

$$E_1 = c\sqrt{p_1^2 + m_1^2 U_1^2} \tag{136}$$

And superluminal particle of $\left|t_2\right\rangle$ with momentum

$$p_2 = m_2\Gamma_2 U_2 \tag{137}$$

is the energy eigenstate with eigenvalue

$$E_2 = c\sqrt{p_2^2 + m_2^2 U_2^2} \tag{138}$$

Let it be supposed that a superluminal particle with a different flavor is generated at the source. It will be produced as a linear combination of two states with different mass eigenstates. The oscillation probability of finding flavor state $\left|t_\beta\right\rangle$ in the case if the initial state is represented by pure $\left|t_\alpha\right\rangle$ beam, is found from the relation, in accordance with neutrino physics [1-3]

$$\begin{aligned} P\left(\left|t_\alpha\right\rangle \to \left|t_\beta\right\rangle\right) &= \left|\left\langle t_\beta(x,t)\middle|t_\alpha(0,0)\right\rangle\right|^2 \\ &= \sin^2 2\theta \sin^2\left(\frac{\phi_1-\phi_2}{2}\right) \end{aligned} \tag{139}$$

And again of finding $\left|t_\alpha(x,t)\right\rangle$ at the oscillation distance x the appropriate probability will be:

$$\begin{aligned} P\left(\left|t_\alpha\right\rangle \to \left|t_\alpha\right\rangle\right) &= 1-\left|\left\langle t_\beta(x,t)\middle|t_\alpha(0,0)\right\rangle\right|^2 \\ &= 1-\sin^2 2\theta \sin^2\left(\frac{\phi_1-\phi_2}{2}\right) \end{aligned} \tag{140}$$

In the case of equal energy $E_1 = E_2 = E$ assumption, the phase difference is equal

$$\begin{aligned} \left(\frac{\phi_1-\phi_2}{2}\right) &= \frac{1}{2\hbar}\left[\left(p_1x-E_1t\right)-\left(p_2x-E_2t\right)\right] \\ &= \frac{1}{2\hbar}\left(p_1-p_2\right)x = \frac{1}{2\hbar}\left(p_1-p_2\right)\frac{U_1+U_2}{2}t \\ &= \frac{1}{2\hbar}\left(p_1-p_2\right)\bar{U}t \end{aligned} \tag{141}$$

The oscillation length is found from the relation

$$P\left(t_\alpha(0,0) \to t_\alpha(L,T)\right) = 1 \tag{142}$$

From here, it is found:

$$\frac{\phi_1-\phi_2}{2} = \pi \tag{143}$$

i.e.

$$\frac{p_1-p_2}{2\hbar}L = \pi \tag{144}$$

From this, on the basis of (139), one sees that regardless of the fact which momentum is larger or smaller, one can write

$$\left(p_1-p_2\right)L = h \tag{145}$$

where L is a distance – length of the oscillation (from the source to the detector). On that path, during the motion through a vacuum, the process of oscillating occurs, which implies turning of one flavor state into another flavor state, and vice versa, according to the scheme $\left(t_\alpha \to t_\beta \to t_\alpha\right)$. Here, h represents the Planck's constant $\left(h = 6{,}62x10^{-34}\,Js\right)$.

If the relations for energy (136) and (138) are introduced, with the assumption that their energies are equal, the following connection between the squares of their momentums is found

$$p_1^2 - p_2^2 = m_2^2U_2^2 - m_1^2U_1^2 \tag{146}$$

The oscillation process is viewed under ultra-relativistic speeds when it can be considered with sufficient precision that:

$$\frac{m_1^2U_1^2}{p_1^2} << 1; \frac{m_2^2U_2^2}{p_2^2} << 1 \tag{147}$$

Then, the impulses (momentums) can be expressed in the form of approximation expressions:

$$
\begin{aligned}
p_1 &\approx \frac{E}{c} - \frac{m_1^2 c U_1^2}{2E} \\
&= \frac{E}{c}\left(1 - \frac{m_1^2 c^4}{2E^2}\left(1+2\delta_1\right)\right) \\
&= \frac{E}{c}\left(1-\delta_1\left(1+2\delta_1\right)\right) \approx \frac{E}{c}\left(1-\delta_1\right); \\
&\delta_1 << 1; \delta_1^2 \approx 0
\end{aligned} \tag{148}
$$

$$p_2 \approx \frac{E}{c} - \frac{m_2^2 c U_2^2}{2E} = \frac{E}{c}\left(1-\delta_2\right) \tag{149}$$

Changing the approximate values with the impulses (148) and (149) in the equation (145), one gets the equation of oscillations for two superluminal particles:

$$\frac{c}{2E}\left(m_2^2 U_2^2 - m_1^2 U_1^2\right) L = h \tag{150}$$

On the basis of the relation (146), this equation gets the other form:

$$\frac{c}{2E}\left(p_1^2 - p_2^2\right) = \left(p_1 - p_2\right) \tag{151}$$

From this, it stems that the energy of the particles with mass eigenstates $|t_1\rangle$ and $|t_2\rangle$, which participate in the oscillation of flavor states, is approximately equal to the product of the mean value of their impulses and the speed of light:

$$E = \frac{p_1 + p_2}{2} c = \overline{p} c \tag{152}$$

If the relation (145) is written in the form

$$L = \frac{h}{p_1 - p_2} \tag{153}$$

one can see that the oscillation length depends exclusively from the impulse of the particles that participate in that process.

Using the approximation relations (148) and (149), this equation can also be written in the following form:

$$L = \frac{2Eh}{c\left(m_2^2 U_2^2 - m_1^2 U_1^2\right)} = \frac{hc}{E\left(\delta_2 - \delta_1\right)} \tag{154}$$

Or in the form

$$
\begin{aligned}
L &= \frac{hc}{E\left(\dfrac{m_2^2 c^4}{2E^2} - \dfrac{m_1^2 c^4}{2E^2}\right)} \\
&= \frac{2hE}{c^3\left(m_2^2 - m_1^2\right)}
\end{aligned} \tag{155}
$$

The formula for the oscillation length that is already known in the neutrino physics is obtained.

Comment. In the case when $U = c\left(1+\delta\right); \delta << 1$ where δ can be considered to be equal to zero, the expression for energy becomes:

$$
\begin{aligned}
&E = c\sqrt{p^2 + m^2 U^2} \approx c\sqrt{p^2 + m^2 c^2}; \\
&m\left(1+\delta\right) \approx m
\end{aligned} \tag{156}
$$

Let the expression (156) for energy in different domains of speed be analyzed.

Domain $\left(c, \left(1+\delta\right)c\right)$

$$E \approx c\sqrt{p^2 + m^2 c^2} = pU \approx pc\left(1 + \frac{m^2 c^4}{2E^2}\right) \rightarrow$$

$$U = c\left(1 + \frac{m^2 c^4}{2E^2}\right) = c\left(1+\delta\right); \delta << 1$$

Spacetime fabric in this case is

$$\begin{pmatrix} ct_T \\ X \end{pmatrix} = \begin{pmatrix} X - X \bullet \delta \\ X \end{pmatrix} \rightarrow t_T = \frac{X}{U} \tag{157}$$

Domain $\left(0, c\right)$

Formula (156) completely matches the formula from the relativistic physics.

$$E \approx c\sqrt{p^2 + m^2 c^2} = \frac{pc^2}{v} \approx pc\left(1 + \frac{m^2 c^4}{2E^2}\right) \rightarrow$$

$$v = c\left(1 - \frac{m^2 c^4}{2E^2}\right) = c\left(1-\delta\right); \delta << 1$$

Writing the spacetime fabric in this case, one has:

$$\begin{pmatrix} ct_E \\ X \end{pmatrix} = \begin{pmatrix} X + X \bullet \delta \\ X \end{pmatrix} \rightarrow t_E = \frac{X}{v} \tag{158}$$

Note. This is the known formula from the relativistic physics and the obtained velocity is in accordance with the velocity that is expected in a laboratory. Therefore, applying the same, but approximate relation for the energy of the same particle, depending on which speed domain it is applied in, the velocity of a particle is found. As it is the same particle, with the E and impulse p, and if it is analyzed it in the domain $\left(c, \left(1+\delta\right)c\right)$, then it behaves as a superluminal particle with the velocity $U = c\left(1+\delta\right)$. And, if the same particle is observed in the domain $\left(0, c\right)$, then the same particle ostensibly looks as if it was not superluminal any more (158). It ostensibly becomes slower and its measured velocity is $v = c\left(1-\delta\right)$. These quantities are obtained between the detector and the source at the distance X and time t_E through the Lorentz matrix, which is obtained by mapping the tachyon matrix from the domain $\left(c,(1+\delta)c\right)$ into the domain $\left(0,c\right)$ according to the scheme:

$$\begin{pmatrix}\Gamma\beta_T & \Gamma \\ \Gamma & \Gamma\beta_T\end{pmatrix} \to \begin{pmatrix}\gamma & \gamma\beta \\ \gamma\beta & \gamma\end{pmatrix};$$
$$\begin{pmatrix}ct_T \\ X\end{pmatrix} \to \begin{pmatrix}ct_E \\ X\end{pmatrix} \tag{159}$$
$$\beta_T = \frac{U}{c} > 1; \beta = \frac{v}{c} < 1$$

Comment. The usage of the Lorentz matrix, which is mapped according to the scheme (159) on measuring the time of flight of tachyons between the source and detector, inevitably leads to the time paradox, which is related to the impossibility of measuring the velocities higher than the speed of light. However, there is a method of measuring which does not use transformation matrices, which will be discussed in the following section. Then, the time paradox does not occur, therefore, superluminal velocities can be measured.

7.4. Speed Mapping

The first point that can be stated is that there are no rest frames for tachyons as superluminal particles because their velocities are restricted to below the speed of light. Secondly, none of the equipment made of classic material fabric for measuring their velocity can, even mentally, go beyond the light barrier.

The only means that remains to be used for the detection of those particles is a classic laboratory. However, that means that they are observed in the environment which is not natural for them.

This signifies that the particles which belong to the velocity domain $\left(c,c\left(1+\delta\right)\right)$ have to be observed in the domain $\left(0,c\right)$ and the fact should be noted that due to such unnatural observation, some paradoxical results must occur.

The observation of tachyons in the latter domain does not affect their energy and impulse, but it will lead to the change in their velocity.

The observation in the latter domain means that the physical characteristics of tachyons must be mapped from the domain $\left(c,c\left(1+\delta\right)\right)$ into the domain $\left(0,c\right)$.

Thus, the following situation occurs: The expressions for energy of tachyons in the domain $\left(c,c\left(1+\delta\right)\right)$ are:

$$E = c\sqrt{p^2 + m^2U^2} = pU \tag{160}$$

and

$$E = \left(U^2/c\right)\sqrt{p^2 - m^2c^2} = pU \tag{161}$$

The expression for energy (160) is mapped into the domain $\left(0,c\right)$ as

$$E = c\sqrt{p^2 + m_E^2c^2} = m_E\gamma c^2 = \frac{pc^2}{v} \tag{162}$$

As it could be seen, the transition from one domain into the other is followed by the seeming change of the mass of the particle

$$m_E = m(1+\delta) \tag{163}$$

Therefore, in the domain $\left(c,c\left(1+\delta\right)\right)$ the observed mass is m , and in the domain $\left(0,c\right)$ the observed mass is m_E .

If the relation (163) is included into the expression for energy

$$\begin{aligned} E &= c\sqrt{p^2 + m^2U^2} \\ &= c\sqrt{p^2 + m^2c^2\left(1+\delta\right)^2} = c\sqrt{p^2 + m_E^2c^2} \end{aligned} \tag{164}$$

one gets the expression for energy mapped from the domain $\left(c,c\left(1+\delta\right)\right)$ into the domain $\left(0,c\right)$.

By equalizing the expressions (160) and (162) one gets

$$\frac{pc^2}{v} = pU \tag{165}$$

the relation for mapping the velocities. According to this, the velocity of tachyons as measured quantity in the domain $\left(0,c\right)$ would be

$$v = \frac{c^2}{U} = \frac{c}{1+\delta} < c \tag{166}$$

This relation can also be determined on the basis of equivalent algebraic expression:

$$\frac{c-v}{c+v} = \frac{U-c}{U+c} \tag{167}$$

Therefore, a paradoxical result (166) is obtained which shows the impossibility of measuring the speed of superluminal particles.

It turns out that the measured speed of a superluminal particle in a classic laboratory is lower than the speed of light. The reason for that will be discussed in the following section.

Comment. If it was known that it was a superluminal particle, then it would be enough to know its energy and momentum to determine its velocity. In that case, there would be no need for measuring the time of travel over a certain distance, and, thus, its velocity would be equal to the quotient of these two physical quantities:

$$U = \frac{E}{p} = \frac{c^2}{v} = c\left(1+\delta\right) \tag{168}$$

Nevertheless, the crucial question is how to find out if it is a superluminal particle. In order to find the answer to this question, some of the criteria need to be presented which could be used to decide whether a particle possesses the characteristics of a tachyon. First, it is necessary for it to be a particle without any electric charge. It means that it cannot be put into the state of motion close to the speed of light by any additional fields. Second, that particle needs to have real mass, as is established by the proposed procedure.

8. Measurement of Tachyon's Velocity

As it has been stated above, there is no rest reference frame attached to the tachyon world lines. Thus, in that sense, special relativity does not exactly treat tachyons on the same footing as slower than light particles.

Therefore, theoretical possibilities of measuring the neutrino velocity will be considered. They are based on two possible manners of observing superluminal particles.

In order to explain those manners, the negative feedback amplifiers in electronics could be mentioned. With the negative feedback factor labeled by β, it acts on the output signal of the amplifier by reducing it. And, by that, the amplification of the signal is reduced, given by the relation between the output and input signal, through the known formula for the amplification of the input signal. Therefore, it is useful to remember that formula.

The gain of the amplifier with feedback called the closed-loop amplifier, A_{FB} is given by

$$A_{FB}=\frac{v_{out}}{v_{in}}=\frac{A_0}{1+\beta A_0} \tag{169}$$

where β is feedback factor which governs how much of the output signal is applied to the input; A_0 is the open-loop gain when $\beta=0$. The application of this formula on the case of measuring neutrino velocity will be used in the following manner: it will be considered that the open-loop gain is $A_0=1$; v_{out} -is the velocity measured in a laboratory; v_{in} - is the natural velocity of neutrinos before entering a laboratory when the measuring process is initiated, thus, the formula adapted to the measuring conditions in the following form can be written

$$\frac{v_{out}}{v_{in}}=\frac{v}{U}=\frac{1}{1+\beta} \tag{170}$$

From here, the feedback factor is found

$$\beta=\frac{U}{v}-1=(1+\delta)^2-1=\frac{1}{\Gamma^2} \tag{171}$$

which is apparently in the function of tachyon matrix elements

$$T=\begin{pmatrix}\Gamma\frac{U}{c} & -\Gamma\\ -\Gamma & \Gamma\frac{U}{c}\end{pmatrix} \tag{172}$$

Thus, the neutrino velocity measured in the laboratory is

$$\begin{aligned}v_{out}=v&=\frac{U}{1+\beta}=\frac{U}{1+\frac{1}{\Gamma^2}}=\frac{U}{1+\frac{1}{\gamma^2}\frac{U^2}{c^2}}\\&=\frac{c^2}{U};\gamma=\Gamma\frac{U}{c};U=c\sqrt{1+\frac{1}{\Gamma^2}}\end{aligned} \tag{173}$$

This measuring method cannot eliminate the feedback factor β in any way, because it influences the measuring results. That is a measuring method which monitors the neutrino flight time over a precisely defined distance, from the source of neutrinos to their registration in the detector. During that monitoring, it is inevitable to use various inertial reference systems over the Lorentz transformation matrix. And, if that is a superluminal particle, as neutrinos are considered to be, then its tachyon matrix T is mapped from the domain $(c,c(1+\delta))$ into the Lorentz matrix L into the domain $(0,c)$, according to the scheme:

$$T=\begin{pmatrix}\Gamma\frac{U}{c} & -\Gamma\\ -\Gamma & \Gamma\frac{U}{c}\end{pmatrix}\to L=\begin{pmatrix}\gamma & -\gamma\frac{v}{c}\\ -\gamma\frac{v}{c} & \gamma\end{pmatrix} \tag{174}$$

The examples which measure the neutrino velocity, monitoring the neutrino time of flight, using the abovementioned transformation matrices, are as follows [4-8]:

MINOS: $\frac{v-c}{c}=(1.0\pm1.1)\times10^{-6}$

OPERA: $-1.8\times10^{-6}<\frac{v-c}{c}<2.3\times10^{-6}$

Borexino: $\frac{|v-c|}{c}<2.1\times10^{-6}$

LVD: $-3.8\times10^{-6}<\frac{v-c}{c}<3.1\times10^{-6}$

ICARUS: $\frac{v-c}{c}=(0.7\pm2.8)\times10^{-7}$

The first thing that can be pointed out is that all measurements in the abovementioned laboratories are mutually independent and that the obtained results for the neutrino velocity almost coincide. In these laboratories, due to the usage of the Lorentz matrix in the measuring process, the formula (170) will be applied.

Therefore, all measured velocities, because of the influence of the feedback through an element of the transformation matrix, are given by the formula

$$v = \frac{U}{1+\beta} = \frac{U}{1+\frac{1}{\Gamma^2}} = \frac{U}{1+\frac{U^2}{c^2}-1} = \frac{c^2}{U} \quad (175)$$

On the other hand, all results of laboratory measurements are expressed in a relative relation with the speed of light through the formula:

$$\frac{v-c}{c} = -\delta \quad (176)$$

From here, one gets

$$v = c(1-\delta) = \frac{c}{1+\delta} < c; \delta << 1 \quad (177)$$

And, of course, the measured velocity of neutrinos is found by the formula:

$$v = \frac{c^2}{U} = \frac{c}{1+\delta} < c \quad (178)$$

that has been stated by all laboratories. Owing to the result (178) obtained by this method, it is clearly seen that it is impossible to measure the speed of superluminal particles, even if they by any chance got in touch with a laboratory. The reason for the occurrence of this result is in the influence of the feedback factor β that is realized through the elements of the tachyon matrix Γ and because of that influence, all laboratories measure the velocity of neutrinos as below the speed of light.

8.1. The Analysis of the Results of Emitted Neutrinos and Photons of Light During the Explosion SN1987A

The second method of measuring neutrino velocity was applied in the detection of neutrinos and photons of light emitted during the explosion of Supernova SN1987A 168,000 light years away [14, 15]. In this case, the detectors in laboratories registered the time moment of the reception of the emitted neutrinos and photons of light. This measuring method does not monitor the time of flight of the emitted particles from the source to the detector over the inertial reference systems that require mandatory usage of transformation matrices. According to that, in this case, the closed-loop gain equals one, because the feed-back factor equals zero $(\beta = 0)$, and the formula (169) is reduced to the form

$$\frac{v_{out}}{v_{in}} = \frac{v}{U} = \frac{1}{1+0} \rightarrow v_{out} = v_{in} = U \quad (179)$$

This measuring method does not have the disrupting factor in the form of a coefficient of tachyon or the Lorentz matrix and, therefore, it should be expected that the measured neutrino velocity is the right one.

During the explosion of Supernova SN1987A 168,000 years ago, neutrinos and photons of light reached the Earth on 23rd February 1987, when they were registered by the detectors of laboratories. According to the reports of those laboratories, published in the same year, neutrinos were detected first and then, three hours later, the photons of light were detected. The following explanation was provided for the delay of the photons of light: Neutrinos arrived 3-4 hours earlier than photons, because photons could not pass through the outer layers of SN1987A before those layers got thin enough. The analysis of those occurrences will be performed and, then, on the basis of the obtained results, a suitable comment will be provided. Thus, the conclusion in contemporary physics is: Even though neutrinos arrived earlier from the distance of 168,000 light years, where the explosion SN1987A occurred, it has been concluded that they are slower than light.

8.2. Case 1. The Assumption That Neutrinos Are Tachyons and That They Are Simultaneously Emitted with Photons

In this case, it is written:

$$\frac{L}{U} + \Delta t = \frac{L}{c} \rightarrow \frac{L}{c(1+\delta)} + \Delta t = \frac{L}{c}; \quad U = c(1+\delta) \quad (180)$$

and from here, one finds:

$$\delta = \frac{U\Delta t}{L} << 1 \quad (181)$$

From here, the following value for δ is found:

$$\delta = \frac{c\Delta t}{L}\frac{1}{1-\frac{c\Delta t}{L}} \approx \frac{c\Delta t}{L}\left(1+\frac{c\Delta t}{L}\right) = \frac{c\Delta t}{L} + \left(\frac{c\Delta t}{L}\right)^2 \approx \frac{c\Delta t}{L} \quad (182)$$

Comment.

Let the estimation for δ be performed with the following parameters:

1. Neutrinos arrived earlier for the time $\Delta t = 3h = 3\times 3600s$
2. Neutrinos had travelled the distance of

$$L = 168\times 10^3 ly = 168\times 10^3 \times 365\times 24\times 86400 \times 3\times 10^8 m$$

The calculated deviation above the speed of light is

$$\delta = \frac{c\Delta t}{L} \approx 10^{-7} \quad (183)$$

That would be approximate to the value obtained by the laboratories [4-8] measuring the velocity of neutrinos:

$$\delta = \left| \frac{v-c}{c} \right| \approx 10^{-7} \tag{184}$$

Taking into consideration (183), one gets the velocity of neutrinos as tachyons

$$U = c(1+\delta) = c\left(1+\frac{c\Delta t}{L}\right) \approx c\left(1+10^{-7}\right) \tag{185}$$

Thus, the measured delay Δt (181) of photons in relation to neutrinos can occur only if neutrinos are superluminal particles.

8.3. Case 2. The Example When Neutrinos Could Be Subluminal

Let it be assumed that photons and neutrinos are simultaneously emitted during the explosion SN1987A, but that photons are delayed and the reason of that delay is said to be: Photons had to wait until the envelope got thin enough to be passed through. They are detained for the time period ∂t until they start moving freely. For that time, neutrinos will travel the way $v\partial t$, and the distance L, where the detector, before photons for the measured interval Δt

$$L - v\partial t = v\left(t_{ph} - \Delta t\right) = v\frac{L}{c} - v\Delta t \tag{186}$$

From here, the photon delay is found

$$\begin{aligned} &\frac{L}{v} - \frac{L}{c} + \Delta t = \partial t \\ &\frac{L}{c} + \delta\frac{L}{c} - \frac{L}{c} + \Delta t = \partial t \rightarrow \\ &\partial t = 2\Delta t \end{aligned} \tag{187}$$

From this result it follows that, in case neutrinos are subluminal, photons will arrive after neutrinos for the measured time of Δt if they get delayed in the explosion process, 168,000 light years away, within SN1987A in the time period of $\partial t = 2\Delta t \approx 6h$.

Comment. The Sun produces photons in its core which need around 50 million years of retention inside it until they reach the outer rim, when they get emitted in the outer space as sunlight.

During the SN1987A explosion, it is said that photons are delayed: Photons had to wait until the envelope got thin enough to be passed through. That detaining of photons, until the envelope got thin enough, according to the given calculation, lasted just $\partial t = 2\Delta t \approx 6h$ which had a consequence that, after the travelled distance of $L \approx 168000ly$ (ly=light year), they got detected later for $\Delta t \approx 3h$ at the detectors on the Earth in comparison to neutrinos.

That long detention of photons during the SN1987A explosion in relation to neutrinos should be the subject of further research, because it is obviously one of the key factors, as a physical phenomenon in defining the nature of neutrinos.

9. Conclusions

In this theory, a tachyon has been considered as a particle which possesses real mass but it is without rest mass.

It should be kept in mind that there is no rest reference frame attached to the tachyon world lines. Therefore, in that sense, special relativity does not really treat tachyons on the same footing as slower than light particles.

Thus, for tachyons as superluminal particles there are no rest frames because their velocities are restricted to below the speed of light. However, the velocities are not restricted to above it and, therefore, the limit of infinite velocities may always be considered.

Based on the propositions of the physics of superluminal particles, the following results obtained on the basis of this theory can be underlined:

There are two points when tachyon energy has a tendency towards the infinite value: the first point is at the speed of light and the second one is at the infinite velocity. The tachyon momentum at the speed of light has a tendency towards the infinite value, but at the infinite velocity its value is definite and equals mc .

As the main conclusion, it could be wondered:

1. Neutrinos as subluminal and independent particles, each on their own, have the phase of final value. However, if neutrinos can be independent, how can it be possible for them to unite – combine and create the flavor state? And, by that, to meet the condition for the flavor state oscillation? If they possess the characteristic to combine – unite, then they are no longer independent and autonomous. By uniting, they lose their independence. However, what makes them unite, if there is such a phenomenon? Do all of them unite or just some of them? And, the ones that unite are no longer autonomous. Thus, their autonomy and their uniting are two features that are mutually exclusive. It is impossible to be autonomous and, at the same time, to lose that autonomy by joining other types of neutrinos. And, if they are just autonomous, then, there is no mismatch between the flavor state and mass states and, according to that, they will not oscillate. Let it be shown how one may theoretically reach the conclusion that neutrinos oscillate. As it has been shown, neutrinos themselves are independent and each type has their own phase. However, when the assumption of the mutual time is introduced, they lose their independence. They mix with each other creating the conditions for the occurrence of the mismatch between mass states and flavor states. Introducing that assumption provides the conditions for the concurrence of the theory and experiment. Neutrinos, which are in the first step autonomous each on their own, with the introduction of the theoretical assumption of mutual time, lose their autonomy. The question arises: to what extent is that theoretical intervention correct, apart from the fact that the obtained results match the experiment?

Now, let the focus shift to tachyons. First, by observing tachyons independently of other particles, each type on its own, they give the result that their phase equals zero. That feature could be interpreted so that tachyons do not exist as independent particles.

However, if tachyons existed, they would appear in nature only and exclusively in a united state. Such a united and combined state would provide the mismatch between flavor states and mass states, which is in accordance with the definition on flavor state oscillations.

In conclusion: Tachyons could not exist in nature as independent particles. And, if they existed, they would appear in a united state. Directly, all three types would mix and combine according to the definition of the mismatch.

Such conclusion is derived on the basis of the theoretical knowledge obtained. The first is that tachyons as independent particles have zero phase and, therefore, could not exist in nature. The second is that, opposite to tachyons, neutrinos as subluminal particles could exist as independent particles. However, that mutual independence of theirs opposes the definition of the occurrence of oscillations. Owing to that, a subsequent theoretical intervention has been introduced in the neutrino physics; neutrinos lose their initial autonomy, due to the need to concur with the experimental results.

However, theoretically observed, tachyons could lose their independence by a simple introduction of mutual time or their average velocity during movement into formulas for phases, by which the conditions are met for their oscillating. Thus, simply put, an autonomous – lone tachyon could not exist in nature and, if it existed, it would appear united only and exclusively together with other types of tachyons. To emphasize once again:

1. Subluminal neutrinos could appear in nature as independent particles

2. Such neutrinos, if they mixed with each other, would not be independent from other types of neutrinos any more. This contradicts the point 1.

3. From the points 1 and 2 it follows that subluminal neutrinos could not mutually oscillate.

4. Subluminal neutrinos, theoretically observed, can be made mutually dependent under a specific theoretical condition, as is done in neutrino physics, in order to obtain the concurrence of theoretical result with experiments.

For tachyons, we could also state the following:

1. In nature, there could not be a lone – autonomous or isolated type of tachyons from other types.
2. If they existed in nature, then, a type of tachyons would necessarily appear united with other types of tachyons.
3. The point 2 is in the direct accordance with the definition on flavor state oscillations.

Therefore, observed theoretically, on the basis of overall considerations, it could be concluded that the theoretical result under the point 2 occurred not by introducing a separate theoretical condition, but spontaneously. It is said spontaneously because the appearance of one type of tachyons in nature would be exclusively related to their uniting with other types.

Comment. On the basis of theoretical considerations, it is necessary to heed the following:

1. One type of tachyons could not exist in nature on its own, lone and isolated from other types of tachyons.

2. One type of subluminal neutrinos could appear in nature independent of other types of neutrinos.

3. A tachyon, if it existed in nature, would appear united with other types of tachyons.

Thus, observing just points 2 and 3, it could be concluded:

On the basis of the point 2, the feature of subluminal neutrinos would not be in accordance with the definition of flavor state oscillations, whereas, on the basis of the point 3, the feature of tachyons would be in accordance with that definition.

It has been shown that it is impossible to measure the velocities higher than the speed of light in certain circumstances. A special attention has been devoted to the method of measuring the velocity of neutrinos in contemporary laboratories, as well as to the analysis of the results of the arrival of neutrinos and photons in the laboratories on the Earth, emitted during the explosion of Supernova SN1987A.

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